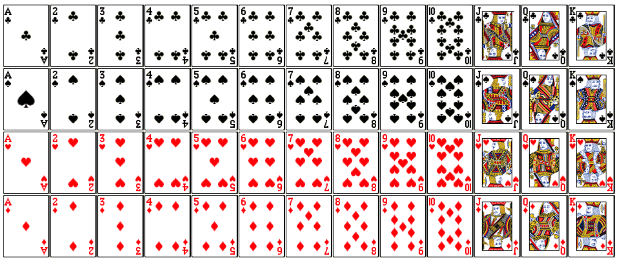
12.3 Permutations and Combinations



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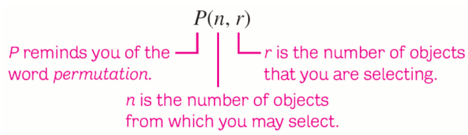
<u>6</u> x <u>5</u> x <u>4</u> x <u>3</u>

= 360

Permutations

DEFINITION A **permutation** is an ordering of distinct objects in a straight line. If we select r *different* objects from a set of n objects and arrange them in a straight line, this is called a *permutation of n objects taken r at a time.* The number of permutations of n objects taken r at a time is denoted by $P(n, r)$.



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P(6, 4) =

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Permutations

- Example: How many permutations are there of the letters *a, b, c, d, e, f*, and *g* if we take the letters three at a time? Write the answer using $P(n, r)$ notation.

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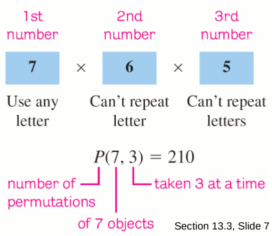
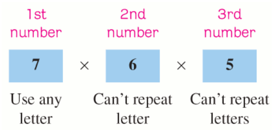
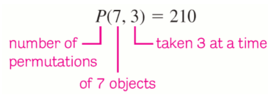


P(n,r) describes a slot diagram.

n = number in first slot
r = number of slots

<u>n</u>	<u>(n-1)</u>	<u>(n-2)</u>	<u>(n-3)</u>	…	<u>(last #)</u>
1st	2nd	3rd	4th		rth





How many ways are there to arrange 5 books on a bookshelf?

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P(5,5) =

<u>5</u> x <u>4</u> x <u>3</u> x <u>2</u> x <u>1</u>

= 120

Shortcut/Defintion

DEFINITION If *n* is a counting number, the symbol *n*!, called *n factorial*, stands for the product $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \cdots \cdot 2 \cdot 1$. We define 0! = 1.

Example: 5! = 5x4x3x2x1

Example:

Compute (5 - 2)!

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(5-2)! = 3! = 3x2x1 = 6

Factorial Notation

- Example: Compute $\frac{8!}{5!3!}$.

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- Solution:

$$\frac{8!}{5!3!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1} = 8\cdot 7 = 56.$$

Cancel 5!. Cancel 3!, which equals 6.

Factorial Notation

FORMULA FOR COMPUTING $P(n, r)$

$$P(n, r) = \frac{n!}{(n-r)!}$$

There are 6 people who want to use an elevator. There is only room for 4 people. How many ways can 6 people try to fill this elevator (one at a time)?

P(6, 4) =

= 360

When we care about the order of objects, like books on a bookshelf, we have a permutation.

When we do not care about the order of objects, like 2 people wining a raffle, we have a combination.

Combinations

FORMULA FOR COMPUTING $C(n, r)$ If we choose r objects from a set of n objects, we say that we are forming a **combination** of n objects taken r at a time. The notation $C(n, r)$ denotes the number of such combinations.[†] Also,

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r! \cdot (n-r)!}.$$

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Order does not matter = combination.

$$C(4,2) = \frac{4!}{(4-2)!\ \ 2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6$$

Combinations

- Example: How many three-element sets can be chosen from a set of five objects?
- Solution: Order is not important, so it is clear that this is a combination problem.

$$C(5, 3) = \frac{5!}{3! \cdot (5-3)!} = \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot 2 \cdot 1} = \frac{20}{2} = 10$$

Combinations

- Example: How many four-person committees can be formed from a set of 10 people?

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- Solution: Order is not important, so it is clear that this is a combination problem.

$$C(10, 4) = \frac{10!}{4! \cdot (10-4)!} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 210$$

Example: At a vation spot there are 7 sites to visit, but you only have time for 5. How many different combinations do you have to choose from?

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Order does not matter = combination.

C(7,5) = 21

Combining counting methods.

Sometimes you will have more than one counting idea to find the total number of possibilities.

Example:
2 men and 2 women from a firm will attend a conference. The firm has 10 men and 9 women to choose from. How many group possibilities are there?

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1^{st} task : choose 2 men from 10

2^{nd} task : choose 2 women from 10

Use a slot diagram $\underline{\quad}$ x $\underline{\quad}$
1^{st} 2^{nd}

Combining Counting Methods

Stage 1: Select the two women from the nine available. $C(9, 2) = \frac{9!}{2!7!} = 36$ ways

Stage 2: Select the two men from the ten available. $C(10, 2) = \frac{10!}{2!8!} = 45$ ways

Thus, choosing the women and then choosing the men can be done in $36 \cdot 45 = 1{,}620$ ways.

Example:
How many different outcomes are there for rolling a die and then drawing 2 cards from a deck of cards?

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1^{st} task : roll a die

2^{nd} task : draw 2 cards from 52
(order does not matter)

Use a slot diagram $\underset{1^{st}}{\_\_} \times \underset{2^{nd}}{\_\_}$

Example:
How many different outcomes are there for rolling a die and then drawing 2 cards from a deck of cards?

1^{st} task : roll a die = 6 ways

2^{nd} task : draw 2 cards from 52 = C(52,2)
(order does not matter)

Use a slot diagram $\underset{1^{st}}{\underline{6}} \times \underset{2^{nd}}{\underline{1326}} = 7956$